Time-Domain Full Waveform Inversion based on high order discontinuous numerical schemes

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Seismic Acquisitions

Seismic Acquisitions (Fig.1) are used to get information from the subsurface. This data is in the form of traces collected by the **Receivers**. The traces are representing the evolution of a disturbance (pressure, displacement, constraint, etc) over time. Those curves are revealing the different reflectors of the media through which the wave generated by the **Source** passed.
The objective of the FWI is to retrieve the characteristics of the propagation medium using the data collected during **Seismic Acquisition** campaigns [1].

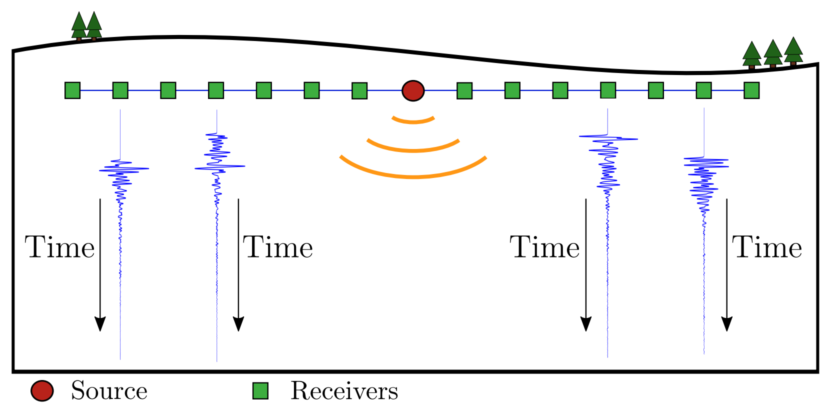


Figure 1: Seismic Acquisition

Wave Propagation Modeling

Continuous problem :
In fluid domain, the propagation of waves is driven by the acoustic wave equation and depends on the nature of the medium. We consider the time domain formulation :

$$\begin{cases} \dfrac{1}{\rho c^2}\dfrac{\partial p}{\partial t} + \nabla\cdot\boldsymbol{v} = f_p \quad \text{on } \boldsymbol{\Omega} \\ \rho\dfrac{\partial \boldsymbol{v}}{\partial t} + \nabla p = 0 \quad \text{on } \boldsymbol{\Omega} \\ p = 0 \quad \text{on } \boldsymbol{\Gamma_1} \\ \dfrac{\partial p}{\partial t} + c\nabla p\cdot\boldsymbol{n} = 0 \quad \text{on } \boldsymbol{\Gamma_2} \\ p(0) = 0, \quad \boldsymbol{v}(0) = 0 \end{cases}$$

With :

p = pressure c = velocity of the media
$\boldsymbol{v}$ = wavespeed ρ = density of the media

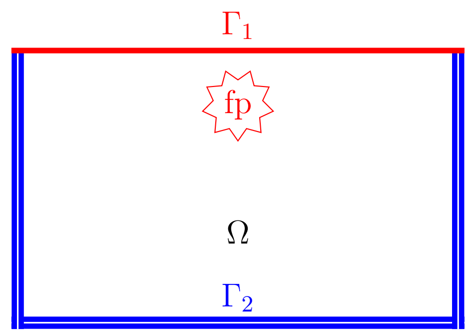


Figure 2: Domain with Absorbing Boundary Conditions

To minimize the effect of an abrupt encapsulation of a finite domain, we can use Absorbing Boundary Conditions (ABC) on $\boldsymbol{\Gamma_2}$ (Fig.2). This boundary condition reduces the computational domain and avoid producing artificial reflections at the boundary.

Time schemes :
To approach the time derivative of the continuous equation we use different explicit time schemes :

- Runge Kutta 2 / 4
- Adam Bashforth 3

Discontinuous Galerkin Method (DGM) :
DGM [2] are still different from the Finite Element Method (FEM) because of the discontinuity of the basis function through the boundaries. Leading to have independent elements that are using fluxes to exchange the numerical information.

DGM Assets :

- Unstructured mesh

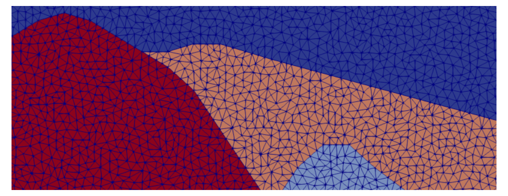


Figure 3: Unstructured mesh adapted to the model

- hp-adaptivity

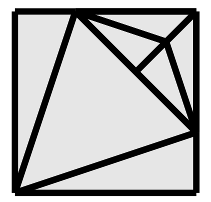


Figure 4: h-adaptivity illustration

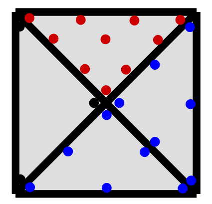
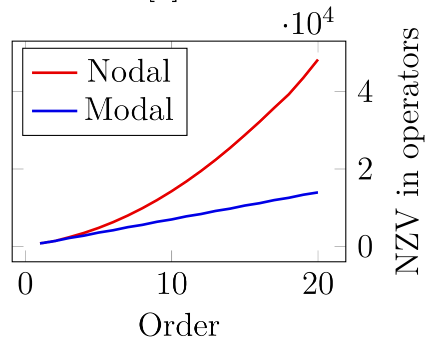


Figure 5: p-adaptivity illustration

- High Performance Computing properties :

Figure 6: 2D mesh partition (10 processors)

- Different Polynomial Basis Function [3]
  - **Nodal** (Lagrange Polynomial basis)
  - **Modal** (Bernstein-Bézier Polynomial basis)



Full Waveform Inversion

The Full Waveform Inversion is a minimisation problem that aim to reconstruct the subsurface parameters $\boldsymbol{m}$ (c, ρ, etc) by using the experimental data collected (d_{obs}). To quantify the differences between the observed data and the current model parameters $\boldsymbol{m}$ under study, we introduce the least-square misfit function defined by :

$$J(\boldsymbol{m}) = \frac{1}{2}||d_{obs} - \mathcal{F}(\boldsymbol{m})||^2$$

That is comparing the experimental data (d_{obs}) with the result obtain with a Forward simulation $\mathcal{F}$ for the current model $\boldsymbol{m}$. The goal of the FWI is to find the optimal $\boldsymbol{m}$ that minimize J.

$$FWI \Leftrightarrow \min_{\boldsymbol{m}}(J(\boldsymbol{m})) \Rightarrow \partial_{\boldsymbol{m}} J(\boldsymbol{m}) = 0$$

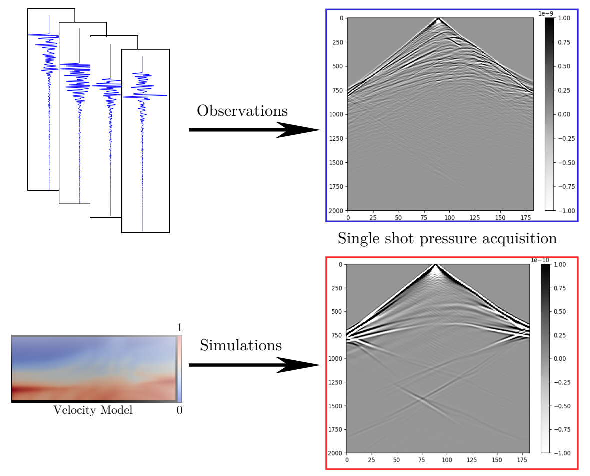


Figure 7: Comparison Observations / Simulations

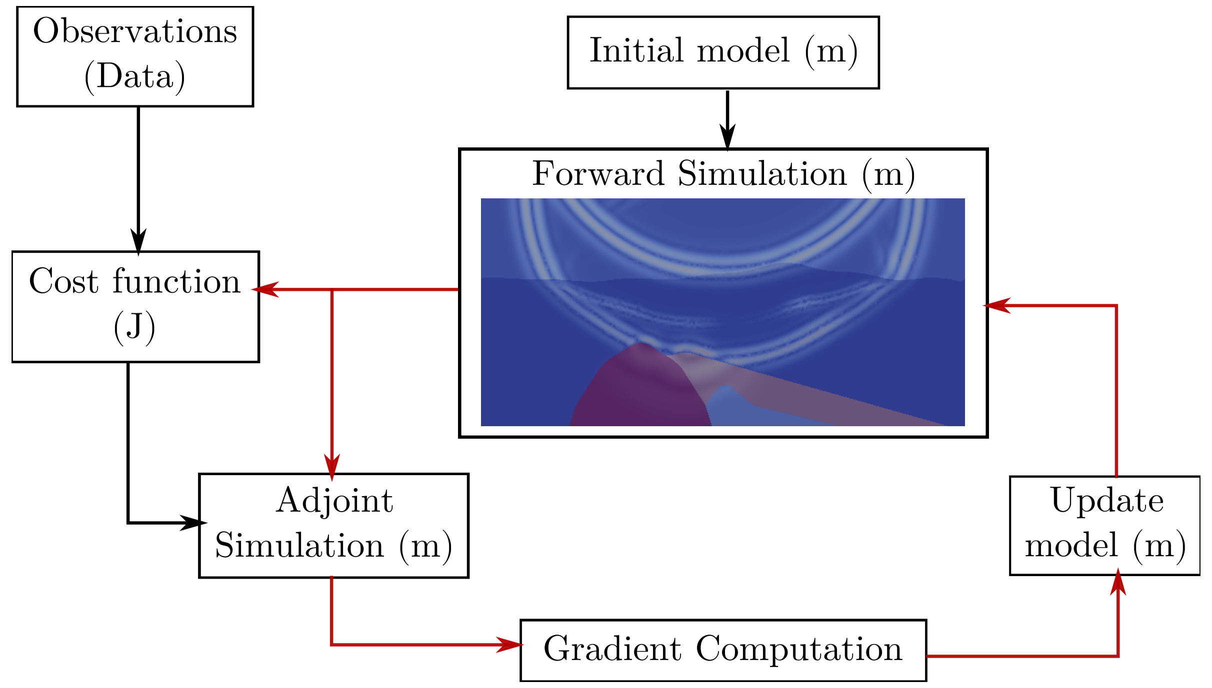


Figure 8: Full Waveform Inversion workflow

The Adjoint State Method :

The FWI is following an iterative process that updates $\boldsymbol{m}$ following a descent direction. This direction needs the computation of the gradient of J by $\boldsymbol{m}$ (Fig.8). This gradient is computed by an **adjoint state method**, which is recommended due to the high amount of parameter to reconstruct [4].

Let us introduce the Lagrangian fonctional :

$$\boldsymbol{\mathcal{L}}(\hat{\boldsymbol{u}}, \hat{\boldsymbol{\lambda}}, \boldsymbol{m}) = \frac{1}{2}||d_{obs} - \mathcal{R}(\hat{\boldsymbol{u}})||^2 + < Forward_{\boldsymbol{m}}(\hat{\boldsymbol{u}}) - f, \hat{\boldsymbol{\lambda}} >$$

With :

_ $\hat{\boldsymbol{u}}$ = Arbitrary wavefield state.
_ $\hat{\boldsymbol{\lambda}}$ = Arbitrary adjoint wavefield state.
_ $\mathcal{R}$ = Wavefield restriction to the receivers
_ $Forward_{\boldsymbol{m}}$ = Left Hand Side of the Forward system.

If $\hat{\boldsymbol{u}} = \boldsymbol{u}$ Solution of ($Forward_{\boldsymbol{m}}(\boldsymbol{u}) - f = 0$) :

$$J(\boldsymbol{m}) = \boldsymbol{\mathcal{L}}(\boldsymbol{u}, \hat{\boldsymbol{\lambda}}, \boldsymbol{m})$$

Let us choose $\hat{\boldsymbol{\lambda}} = \boldsymbol{\lambda}$ such as $\frac{\partial \boldsymbol{\mathcal{L}}}{\partial \boldsymbol{u}} = 0$

$$(\mathcal{R}^* d_{obs} - \boldsymbol{u}) + Forward^*_{\boldsymbol{m}}(\boldsymbol{\lambda}) = 0$$

For $Forward_{\boldsymbol{m}}(\boldsymbol{u}) - f = 0$:

$$\partial_{m_i} J(\boldsymbol{m}) = \partial_{m_i}\boldsymbol{\mathcal{L}}(\boldsymbol{u}, \boldsymbol{\lambda}, \boldsymbol{m}) = \partial_{m_i} < Forward_{\boldsymbol{m}}(\boldsymbol{u}), \boldsymbol{\lambda} >$$

For the following results we aim to reconstruct the acoustic velocity model from the marmousi dataset.

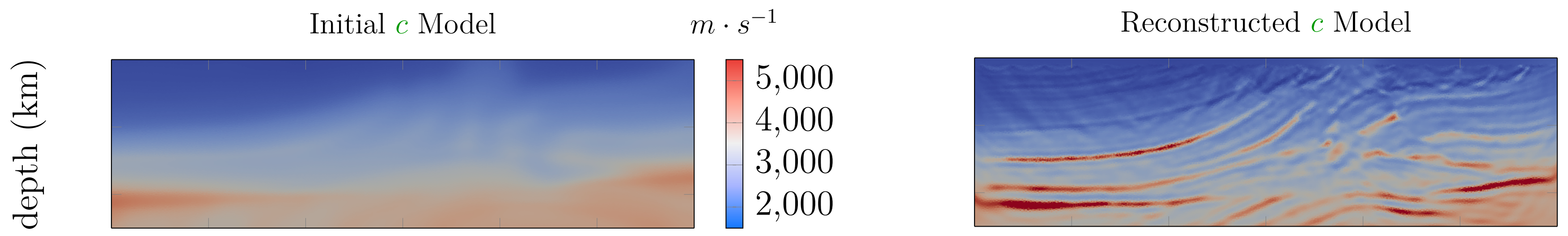
We will consider :

- Constant model par element
- 47k elements
- $\rho = 1$ on Ω
- Synthetic data set noised with a SNR = 10
- Parametrization : $(\frac{1}{\kappa}, \rho)$ (With : $\kappa = \rho c^2$)

Leading to have this gradient expression :

$$\partial_{\frac{1}{\kappa}} J = < \partial_t p, \boldsymbol{\lambda}_p >$$

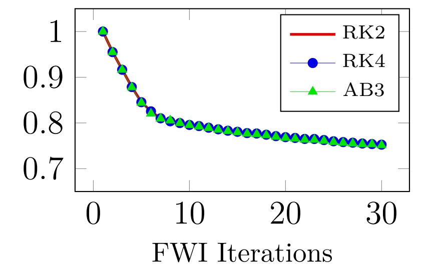
2D Reconstruction



Computational specifications :

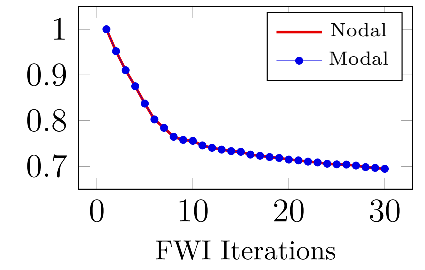
- 47k P1 elements
- Time schemes : RK2, RK4, AB3
- Polynomial basis : Nodal, Modal
- 30 FWI iterations
- 120 cores
- 19 Sources / 181 Receivers

Cost function evolution :



CPU time (Nodal) : 3h15 / 4h30 / 5h10

Cost function evolution :

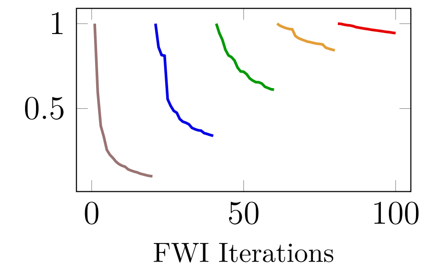


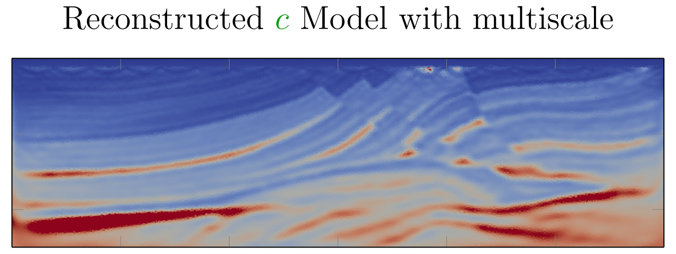
CPU time (RK2) : 3h15 / 4h30

Multiscale Reconstruction [5]

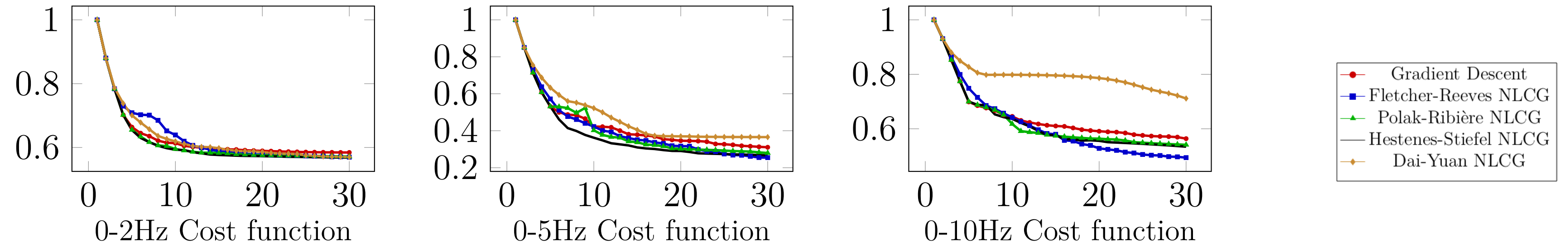
- Time Scheme : RK2
- 120 cores
- 20 FWI iterations per filter
- Computation time : 10h
- Frequencies : 1-2.5Hz, 1-5.0Hz, 1-7.5Hz, 1-10Hz, 1-15Hz

Cost function evolution :





Optimization [6]



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